

Electric field in spontaneously polarized helium-II and a possible “spherical blackout”

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Аннотация

A general method of description of a spontaneously polarized isotropic dielectric is constructed. It is based on the Maxwell equations for a medium and on the statistical averaging of the sources of spontaneous polarization (dipoles or multipoles). We show that the sources of spontaneous polarization in the Maxwell equations should be considered as conditionally foreign charges. In the literature, two basic approaches to the description of spontaneously polarized He II are available: phenomenological and microscopic ones, which give quite different results. Our analysis allows us to make them consistent with each other. In addition, we have found a solution for the electric field in spontaneously uniformly polarized He II placed in a spherical conductor. It turns out that the conductor strongly suppresses the field \mathbf{E} arising in He II due to the spontaneous polarization. Apparently, it is a specific property of a spherical conductor. We have also discussed the experiments with the second sound by Rybalko and Chagovets.

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1 Introduction

It was found in the experiments [1, 2, 3, 4] that the standing half-wave of the second sound in He II induces an electric signal, which is synchronous with the second sound. Though the atoms of He II have no intrinsic dipole or multipole moment, the electric signal *can* appear in electrically neutral helium because of two following properties. First, two atoms of helium slightly polarize each other [5, 6, 7] (tidal polarization). Due to this property, the gravity force, acceleration, density gradient, and temperature gradient cause the bulk spontaneous polarization of helium-II [8, 9, 10, 11]. Second, a nonpolar atom located near the conductor is polarized due to the interaction with it [12]. This property underlies the surface spontaneous polarization of helium-II. A lot of attempts to explain the Rybalko’s effect [1] were undertaken. Till now, the community is separated into two camps. Conditionally, they can be called “microscopic” and “phenomenological” ones. In the first, the authors base themselves on the semimicroscopic approach and conclude that the effect is, most likely, a surface one [13, 14, 10, 15]. In the second camp, the authors use semiphenomenological methods and make a different conclusion that the effect is a bulk one [16, 8, 17, 18, 19]. Moreover, different initial equations are applied in different approaches even for the description of identical processes. In some works, the electric properties of He II were studied, but a model of the Rybalko’s effect was not constructed [20, 21, 22, 23]. In what follows, we construct a method of description of the spontaneous polarization of an isotropic dielectric that joins the phenomenological and microscopic approaches. Note that the known to us books on the electrodynamics of continuous media do not contain a similar method, though the majority of mechanisms of polarization of dielectrics are spontaneous.

2 Description of a spontaneously polarized dielectric

It is known from the electrodynamics of continua (see [24], Chapters II, IV, IX) that the electromagnetic field of isotropic dielectrics is described by the local (i.e., valid at each point of a medium) relations

$$\mathbf{E} = \mathbf{D} - 4\pi\mathbf{P}, \quad (1)$$

$$\mathbf{D} = \varepsilon\mathbf{E}, \quad (2)$$

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} = \mu\mathbf{H} \quad (3)$$

and by the Maxwell equations in a medium:

$$\text{div}\mathbf{D} = 4\pi\rho_f, \quad (4)$$

$$\text{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad (5)$$

$$\text{div}\mathbf{B} = 0, \quad (6)$$

$$\text{rot}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{4\pi\mathbf{j}_f}{c}. \quad (7)$$

Here, \mathbf{E} is the real electric field in the medium, \mathbf{P} is the polarization of the medium that arises as the response to the field \mathbf{D} , \mathbf{D} is the electric induction (if the medium is absent, then the field \mathbf{E} takes the value \mathbf{D}), c is the velocity of light in vacuum, ρ_f is the density of foreign charges, and \mathbf{j}_f is the foreign current. The interconnection between the magnetic field \mathbf{H} , magnetization \mathbf{M} , and magnetic induction \mathbf{B} is similar to the interconnection between \mathbf{E} , \mathbf{P} , and \mathbf{D} .

Equations (1)–(7) are general and are true under any conditions (if the fields are weak) for the spontaneous polarization and for the polarization induced by an external field.

If a dielectric is surrounded by vacuum with a uniform field \mathbf{E}_0 , then the field inside the dielectric is usually also uniform. This field is codirected with \mathbf{E}_0 ($\mathbf{D} = \mathbf{E}_0$, $\mathbf{E} = \mathbf{E}_0/\varepsilon$) for some shapes of a dielectric and is not for other shapes. But, apparently, the field \mathbf{E} is uniform for any shape of a dielectric. The atoms of the dielectric are slightly polarized against the field. Therefore, the polarization $-4\pi\mathbf{P}$, which weakens the external field \mathbf{E}_0 , arises in the medium. However, the spontaneous polarization of a dielectric arises not due to an external field, but due to the internal “nonelectric” mechanism causing the appearance of a set of dipoles (or multipoles) in the medium; we will call them “initial” dipoles. How can we take this specific feature into account in Eqs. (1)–(7)? There are only two possibilities: to consider the initial dipoles as bound charges (and to include them in \mathbf{P}) or as foreign charges (and to include them in ρ_f and \mathbf{j}_f). Let us consider both possibilities.

Let the mean dipole moment of the system of initial dipoles per unit volume be equal to \mathbf{P}_0 . For He II, $\varepsilon \approx 1.057$. The polarizability is determined by the value of $\frac{\varepsilon-1}{4\pi}$, according to the relation

$$\mathbf{P} = \frac{\varepsilon-1}{4\pi}\mathbf{E}, \quad (8)$$

and is very small for He II. If an external field \mathbf{E}_0 is applied to He II, then a small (relative to \mathbf{E}_0) polarization $\frac{\varepsilon-1}{4\pi}\mathbf{E} = \frac{\varepsilon-1}{4\pi\varepsilon}\mathbf{E}_0$ arises in helium. It weakens \mathbf{E}_0 to $\mathbf{E} = \mathbf{E}_0/\varepsilon$. Now, let helium be polarized spontaneously. Namely, let the codirected initial dipoles characterized by the own polarization \mathbf{P}_0 be uniformly distributed in helium. If we identify \mathbf{P} in (1), (2) with \mathbf{P}_0 , then $\mathbf{E} = \frac{4\pi}{\varepsilon-1}\mathbf{P}_0 \approx 220\mathbf{P}_0$. In other words, the electric field \mathbf{E} arising due to the polarization \mathbf{P}_0 turns out to be much larger than the source (the value of \mathbf{P}_0)! It looks like Baron Munchausen charged a cannon with a cannonball of diameter 10 cm, whereupon the

cannon shot off the cannonball of diameter 20 meters. In physics, such situations are met in complex nonlinear systems. In our case, we see no reason for a similar effect. Hence, the spontaneous polarization \mathbf{P}_0 and the polarization \mathbf{P} from (1), (2) have different physical meanings, and we should not set $\mathbf{P} = \mathbf{P}_0$.

In a number of works, it is considered that the relation $\mathbf{D} = 0$ (i.e., $\mathbf{E} + 4\pi\mathbf{P} = 0$) should be satisfied inside a dielectric surrounded by a conductor. It is difficult to agree with it. Since this is important point, let us consider it in more details (see [24], Chapt. I, Sections 1 and 3; Chapt. II, Sections 6 and 7). As is known, the equilibrium inside the conductor means the absence of a current. It requires $\mathbf{E} = 0$ inside the conductor. The induction \mathbf{D} is introduced only for dielectrics, because this quantity involves the polarization of a medium, as the response to the external field \mathbf{E}_0 . If a metal (conductor) is placed into an external field, then its equilibrium will be broken, and the current will flow, until the surface polarization has compensated \mathbf{E}_0 . Therefore, the introduction of \mathbf{D} makes no sense for a metal. However, it turns out (see [25] Chapt. IV, Section 6; [24] Chapt. II, Section 7 and Problem 1 after Section 7) that if we calculate the field outside a dielectric (with some ε) and then set $\varepsilon = \infty$ in the formulae, we get the field outside the *conductor* of the same shape (as that of a dielectric). In this respect, the conductor can be considered as a dielectric with $\varepsilon = \infty$. In this case, we have $\mathbf{E} = 0$ and $\mathbf{D} = \varepsilon\mathbf{E} \neq 0$ in the conductor. On the dielectric-dielectric boundary, the relations $D_{1n} = D_{2n}$ and $E_{1t} = E_{2t}$ hold. On the dielectric-metal boundary, the other relation is valid: $\varphi = \text{const}$ ($\text{const} = 0$, if the metal is grounded), which is equivalent to two relations: $E_{1t} = E_{2t} = 0$ and $D_{1n} = 4\pi\sigma_2$ (1 — the dielectric, 2 — the conductor). If the electricity sources are located in a dielectric, they induce the surface charges with density σ_2 . Their field can be exactly modeled by the introduction of “images” for the charges in the dielectric; the account for σ_2 (or images) allows one to satisfy the condition $\varphi = \text{const}$ on the surface. The conclusion is that, in the presence of the spontaneous polarization in a dielectric, the sources of this polarization create the field $\mathbf{E} \neq 0$ outside themselves. Hence, according to (1) and (2), $\mathbf{D} = \varepsilon\mathbf{E} \neq 0$ in the dielectric. On the dielectric-conductor boundary, we get $D_{1n} = 4\pi\sigma_2 \neq 0$. Thus, for a spontaneously polarized dielectric, the equality $\mathbf{D} = 0$ inside the dielectric is impossible.

We now prove strictly that $\mathbf{P} \neq \mathbf{P}_0$. Let us apply the operation *rot* to Eqs. (5) and (7) at constant ε and μ . As a result, we obtain the equations

$$\Delta\mathbf{E} - \frac{\mu\varepsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{4\pi}{\varepsilon} \nabla\rho_f + \frac{4\pi\mu}{c^2} \frac{\partial \mathbf{j}_f}{\partial t}, \quad (9)$$

$$\Delta\mathbf{H} - \frac{\mu\varepsilon}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = -\frac{4\pi}{c} \text{rot}\mathbf{j}_f. \quad (10)$$

If $\mathbf{P} = \mathbf{P}_0$, then the initial dipoles are taken into account in \mathbf{P} . There are no other foreign charges or currents. Therefore, $\rho_f = 0$ and $\mathbf{j}_f = 0$. In this case, Eqs. (9) and (10) are ordinary wave equations for the free fields \mathbf{E} and \mathbf{H} . Their solutions are the electromagnetic waves with the velocity $\tilde{c} = c/\sqrt{\varepsilon\mu}$. They include a wave, for which $\lambda/2$ is equal to the resonator length L (like in the experiments [1, 2, 3]). However, this wave is characterized by the frequency $\nu = \tilde{c}\lambda$, which is larger by 6 orders of magnitude than the second-sound frequency. Among the solutions there is no wave, whose frequency and wavelength correspond to those of the second sound or other wave process with a velocity different from the velocity of light in the medium (\tilde{c}). It is natural. Indeed, in the absence of foreign charges and currents, the electromagnetic field in the medium is a set of electromagnetic waves moving with the velocity \tilde{c} . But the experiments have registered the electric wave with the second-sound velocity. Hence, $\mathbf{P} \neq \mathbf{P}_0$.

It is easy to show that the initial dipoles should be considered as conditionally foreign charges. Let a dielectric have a small spherical cavity of radius R , and let a foreign charge

q_0 be placed at its center \mathbf{R}_1 . If R is less than the interatomic distance, we can neglect the presence of the cavity and consider that the charge is placed directly in the dielectric. In this case, the field in the dielectric is described by the Eq. (4):

$$\text{div}\mathbf{D}(\mathbf{r}) = 4\pi\rho_f(\mathbf{r}) = 4\pi q_0\delta(\mathbf{r} - \mathbf{R}_1). \quad (11)$$

With regard for the cavity, the charge q_0 should be considered as a source of the external field. Then the field in the dielectric can be determined from the equation $\text{div}\mathbf{D} = 0$ and the boundary conditions on the surface of the spherical cavity: $D_{1n} = D_{2n}$ and $E_{1t} = E_{2t}$ (see [24], Chapt. II). Both ways give the solution $\varphi(\mathbf{r}) = \frac{q_0}{\varepsilon|\mathbf{r}-\mathbf{R}_1|}$ that is the potential induced in the dielectric by a foreign charge q_0 . Let now the charge $-q_0$ be placed in the cavity at a small distance δR from the charge q_0 . Then Eq. (11) should be replaced by

$$\text{div}\mathbf{D} = 4\pi\rho_f = 4\pi q_0\delta(\mathbf{r} - \mathbf{R}_1) - 4\pi q_0\delta(\mathbf{r} - \mathbf{R}_2), \quad (12)$$

where $\mathbf{R}_2 = \mathbf{R}_1 + \delta\mathbf{R}$. A solution of such equation is the potential induced in the medium by a foreign dipole: $\varphi(\mathbf{r}) = \frac{q_0}{\varepsilon|\mathbf{r}-\mathbf{R}_1|} - \frac{q_0}{\varepsilon|\mathbf{r}-\mathbf{R}_2|}$. We now pass to spontaneously polarized He II. Let the initial dipoles be distributed in helium. We surround one such dipole with a sphere of small radius R . It is obvious that, irrespective of the microscopic structure of this dipole, the electromagnetic action of the dipole outside the sphere coincides with the action of the above-described foreign dipole with the same dipole moment: $q_0\mathbf{R}_1 - q_0\mathbf{R}_2 = \mathbf{d}_f$. Therefore, the field outside the initial dipoles can be found by considering them as *foreign dipoles*.

This can be seen in another way: In the derivation of the Maxwell equations (4)–(7), not only the charges connected with the other substance, but also the internal charges guided by an external force should be considered foreign. That is, one needs to understand the words “foreign charge” in the broad sense, as a foreign force. This force is foreign relative to the Maxwell equations (4)–(7). Such force is present, if the coordinate, velocity, and magnitude of an initial dipole are governed by other laws. If the dipoles (multipoles) relate to quasiparticles, those are the laws of two-fluid hydrodynamics. If the dipoles are connected with the polarization of atoms due to the acceleration of a medium, gravity force, density gradient in a medium, or electrostriction, then all they are foreign forces relative to (4)–(7). In these cases, the initial dipoles should be considered as conditionally foreign charges.

Thus, the spontaneous polarization of any nature should be taken into account in (4)–(7) as ρ_f and \mathbf{j}_f . If ρ_f varies synchronously with a second-sound wave, $\rho_f = f(z - v_2 t)$, then a solution of Eqs. (9), (10) is an analogous wave (in a superposition with free electromagnetic waves, which can be rejected). The foreign charges are a source of wave processes, whose velocities are different from \tilde{c} . In this case, the polarization of a medium \mathbf{P} is the response of atoms to the field \mathbf{D} and is less than the latter, which corresponds to the meaning of the polarization in formulae (1), (2), and (8).

If the system contains a condensate of any structure, then there are the correlations between atoms (i.e., between electrons and nuclei of different atoms as well). These correlations should be taken into account in ρ_f and \mathbf{j}_f in Eqs. (1)–(7), and in Ψ_n in Eq. (19).

To determine the electric field in a spontaneously polarized dielectric, one should solve Eqs. (1), (2), and (4) with regard for the boundary conditions and the relation

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}. \quad (13)$$

In the experiment by A.S. Rybalko [1], helium was placed in a grounded metallic resonator. Therefore, the boundary condition is as follows: on the internal surface of the

resonator,

$$\varphi = 0. \quad (14)$$

In a stationary problem, the magnetic field is absent: $\mathbf{A} = 0$, $\mathbf{B} = \text{rot}\mathbf{A} = 0$. If the charges are moving, then a current \mathbf{j}_f and a magnetic field arise. The charge moving with a velocity \mathbf{v} creates the potential φ and the vector potential $\mathbf{A} = \varphi\mathbf{v}/c$ [26] (in the immovable reference system). The last relation indicates that the magnetic field is weak for the processes such as the second sound, which are slow as compared with the electromagnetic wave. And we may set in (13) $\partial\mathbf{A}/\partial t = 0$. Therefore, for the stationary and slow processes, Eq. (4) takes the form

$$\Delta\varphi = -4\pi\rho_f/\varepsilon. \quad (15)$$

Let the initial dipoles $\mathbf{d}_f = |q_0|\mathbf{r}_0$ corresponding to the averaged polarization $\mathbf{P}_0 = n_f\mathbf{d}_f$ be uniformly distributed in helium (here, n_f is the concentration of dipoles, $q_0 < 0$). Then the density of foreign charges reads

$$\rho_f = \sum_{j=1}^{N_f} [q_0\delta(\mathbf{r} - \mathbf{r}_j) - q_0\delta(\mathbf{r} - \mathbf{r}_j - \mathbf{r}_0)], \quad (16)$$

where N_f is the number of initial dipoles in helium, and \mathbf{r}_j and $\mathbf{r}_j + \mathbf{r}_0$ are the coordinates of the effective charges q_0 and $-q_0$ of the dipole. We direct the axis z along \mathbf{P}_0 . Then $\mathbf{r}_0 = r_0\mathbf{i}_z$, and $\mathbf{P}_0 = P_0\mathbf{i}_z$. The solution of Eqs. (15), (16) is known from electrostatics:

$$\varphi(\mathbf{r}) = \sum_{j=1}^{N_f} \left[\frac{q_0}{\varepsilon|\mathbf{r} - \mathbf{r}_j|} - \frac{q_0}{\varepsilon|\mathbf{r} - \mathbf{r}_j - \mathbf{r}_0|} \right]. \quad (17)$$

For the points \mathbf{r} far from the initial dipoles ($|\mathbf{r} - \mathbf{r}_j| \gg r_0$), we can make expansion in \mathbf{r}_0 . As a result, we have

$$\varphi(\mathbf{r}) = \sum_{j=1}^{N_f} \frac{-q_0\mathbf{r}_0(\mathbf{r} - \mathbf{r}_j)}{\varepsilon|\mathbf{r} - \mathbf{r}_j|^3} = \sum_{j=1}^{N_f} \frac{\mathbf{d}_f \cdot (\mathbf{r} - \mathbf{r}_j)}{\varepsilon|\mathbf{r} - \mathbf{r}_j|^3}. \quad (18)$$

This formula is quite applicable to the points inside a system of dipoles, if the distance between adjacent dipoles is much larger than the size of a dipole.

In the experiment, the potential $\varphi(\mathbf{r})$ was measured with the help of an electrode during a macroscopic time. For such time interval, a configuration of initial dipoles change a huge number of times. Therefore, we need to find the average (over the measurement time) value of $\varphi(\mathbf{r})$. According to J.W. Gibbs [27], the average over the time can be replaced by the average over the ensemble, and the probability of a realization of the n -th state depends on the energy of the system, E_n , in this state. Quantum statistics gives the following formula for the average over the ensemble [28]:

$$\langle \hat{\varphi} \rangle = \int d\Omega Z^{-1} \sum_n e^{-E_n/k_B T} \Psi_n^* \hat{\varphi} \Psi_n, \quad Z = \sum_n e^{-E_n/k_B T}, \quad (19)$$

where $\hat{\varphi}$ is given by (17) or (18), and $\{\Psi_n\}$ is the complete collection of wave functions of the system. For the most complete description, the operator $\hat{\varphi}$ and the functions Ψ_n should be written in terms of the coordinates of the nucleus and electrons of each atom (taking into account the interaction of particles inside the atom). The formulae will depend on the mechanism of polarization. Note that formula (19) is true for the equilibrium systems. A second-sound wave changes T insignificantly [1, 2, 3]. Therefore, the system is close to the

equilibrium state all the time, and formula (19) is applicable. Such approach is exact, but too complicated. For $T = 0$:

$$\langle \hat{\varphi} \rangle = \int d\Omega \Psi_0^* \hat{\varphi} \Psi_0. \quad (20)$$

An example of such analysis for $T = 0$ can be found in [29].

At high temperatures, or at low temperatures and the quasiclassical behavior of a system of initial dipoles, we can pass to the classical description [27]:

$$\langle \varphi(\mathbf{r}) \rangle \approx Z^{-1} \sum_n e^{-E_n/k_B T} \varphi_n(\mathbf{r}), \quad Z = \sum_n e^{-E_n/k_B T}. \quad (21)$$

Here, $\varphi_n(\mathbf{r})$ is given by formula (18), E_n is the total energy of the system for the n -th configuration, and the index n enumerates all possible *nonequivalent* configurations of the ensemble of dipoles (i.e., all nonequivalent collections of coordinates $\{\mathbf{r}_j\}$). The configurations different from one another only by a permutation of coordinates are equivalent. It is convenient to include the equivalent configurations in sums (21). In this case, we should add the compensating factor $(N_f!)^{-1}$. As a result, we have

$$\langle \varphi(\mathbf{r}) \rangle \approx (N_f!)^{-1} Z^{-1} \int_V d\mathbf{r}_1 \cdots \int_V d\mathbf{r}_{N_f} e^{-E(\mathbf{r}_1, \dots, \mathbf{r}_{N_f})/k_B T} \varphi(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_{N_f}), \quad (22)$$

$$Z = (N_f!)^{-1} \int_V d\mathbf{r}_1 \cdots \int_V d\mathbf{r}_{N_f} e^{-E(\mathbf{r}_1, \dots, \mathbf{r}_{N_f})/k_B T}, \quad (23)$$

where $\varphi(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_{N_f})$ is determined by formula (18), and V is the volume of the system. The energy $E(\mathbf{r}_1, \dots, \mathbf{r}_{N_f})$ is a sum of “nonelectric” and “electric” parts. The former can be approximately considered identical for different configurations, and it comes out from (22). So, E is determined only by the electric part and is the energy of interacting dipoles [26]:

$$E(\mathbf{r}_1, \dots, \mathbf{r}_{N_f}) = \sum_{j < l} \left[\frac{\mathbf{d}_j \mathbf{d}_l}{r_{jl}^3} - \frac{3(\mathbf{d}_j \mathbf{r}_{jl})(\mathbf{d}_l \mathbf{r}_{jl})}{r_{jl}^5} \right], \quad (24)$$

where $j, l = 1, \dots, N_f$, $\mathbf{r}_{jl} = \mathbf{r}_j - \mathbf{r}_l$, and \mathbf{r}_j is the radius-vector of the j -th dipole.

Formulae (22) and (24) are simpler than (19), but are still very complicated. Therefore, we make one more simplification. *Assume* that, for the configurations with a sufficiently uniform distribution of dipoles, the values of E differ from one another by a small value $\delta E \ll k_B T$ and, therefore, can be considered identical and equal to E_0 . Of course, the values of E must be noticeably different from E_0 for the configurations with very nonuniform distribution of dipoles. But the number of such configurations is very small as compared with the number of all configurations. Therefore, we may take $E = E_0$ also for such particular configurations. In this case, we consider the values of $E(\mathbf{r}_1, \dots, \mathbf{r}_{N_f})$ to be equal for all configurations. Then the Gibbs method (22), (23) leads to the ordinary averaging of the potential $\varphi(\mathbf{r})$ over all possible configurations:

$$\langle \varphi(\mathbf{r}) \rangle \approx V^{-N_f} \int_V d\mathbf{r}_1 \cdots \int_V d\mathbf{r}_{N_f} \varphi(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_{N_f}). \quad (25)$$

Substituting expression (18), we get

$$\langle \varphi(\mathbf{r}) \rangle \approx \frac{N_f}{V} \int_V d\mathbf{r}_1 \frac{\mathbf{d}_f \cdot (\mathbf{r} - \mathbf{r}_1)}{\varepsilon |\mathbf{r} - \mathbf{r}_1|^3} \equiv \int_V d\mathbf{r}_1 \frac{\mathbf{P}_0 \cdot (\mathbf{r} - \mathbf{r}_1)}{\varepsilon |\mathbf{r} - \mathbf{r}_1|^3}. \quad (26)$$

In such approximation, the dipoles are considered as independent. Therefore, the averaging is equivalent to the smearing of each dipole uniformly over the mean volume per one dipole.

This is expressed by the simple formula (26), where the integral is taken over the volume of the system, and $\mathbf{P}_0 = n_f \mathbf{d}_f$ is the constant “foreign” polarization.

If the external field makes the distribution of dipoles to be macroscopically nonuniform (with the concentration $n_f(\mathbf{r})$), then we should consider in (22), (23) that the configurations of dipoles with the concentration close to $n_f(\mathbf{r})$ have the same finite value of E , whereas for the remaining configurations $E = +\infty$. It is sufficiently obvious that the same result would be obtained, if we would transfer the inhomogeneity in the potential, i.e., if formula (18) would be written as

$$\varphi(\mathbf{r}) = \sum_{j=1}^{N_f} \frac{n_f(\mathbf{r}_j)}{n_f^{(0)}} \frac{\mathbf{d}_f \cdot (\mathbf{r} - \mathbf{r}_j)}{\varepsilon |\mathbf{r} - \mathbf{r}_j|^3} \quad (27)$$

(where $n_f^{(0)} = N_f/V$), and the energy of *each* configuration would be taken equal in (22), (23). Then relations (22), (23), and (27) yield

$$\langle \varphi(\mathbf{r}) \rangle = \int_V d\mathbf{r}' \frac{\mathbf{P}_0(\mathbf{r}') \cdot (\mathbf{r} - \mathbf{r}')}{\varepsilon |\mathbf{r} - \mathbf{r}'|^3}, \quad \mathbf{P}_0(\mathbf{r}') = n_f(\mathbf{r}') \mathbf{d}_f. \quad (28)$$

It may be that the transitions from (19) to (21) and from (22)–(24) to (26) lead to a significant distortion of the result. This requires a separate study. But we hope for that the simple formulae (26) and (28) are suitable at least qualitatively.

So, in order to determine the electric field in a spontaneously polarized dielectric, we need to find the potential from (26) or (28). Then we can determine the field strength by the formula $\mathbf{E} = -\nabla\varphi$. Additionally, we should take the boundary conditions into account. If the dipoles have different directions, then \mathbf{d}_f should be replaced in formulae (26), (28) by the ordinary mean $\langle \mathbf{d}_f \rangle$. It is significant that the polarizations \mathbf{P}_0 in (26) and \mathbf{P} in (1), (2) are completely different quantities. The quantity \mathbf{P}_0 does not satisfy Eqs. (1), (2).

Thus, we have constructed a method of description of the spontaneous polarization of a dielectric. The solution for the potential created at a point \mathbf{r} by the system of N_f initial dipoles is given by the formula (18). It should be statistically averaged, which leads to formulae (19)–(28). Formula (28) was already used in the microscopic approach [30, 10, 15] (but it was deduced by the ordinary averaging of potential (18) without a statistical analysis). Our method joins the phenomenological and microscopic approaches. Earlier, the medium was considered in a too simple way in the phenomenological approach, and the connection with the Maxwell equations for a medium was not established in the microscopic approach. Therefore, these approaches were not consistent with each other, by the equations and by the results. The analysis above makes them consistent.

3 Calculation of the electric field in spontaneously polarized helium-II placed in a spherical conductor

We now find the electric field in a spontaneously polarized dielectric placed in a spherical metallic shell. Let the spontaneous polarization be bulk and uniform. This problem is simplest from the viewpoint of the polarization field structure and the symmetry of boundaries. Initially, we chose this problem to illustrate the method. But it turned out that the solution is surprising and interesting.

Consider a grounded metallic sphere of radius R_m . Let it contain He II with uniformly distributed over the volume initial dipoles $\mathbf{d}_f = |q_0| \mathbf{r}_0$, corresponding to the mean spontaneous polarization $\mathbf{P}_0 = n_f \mathbf{d}_f$ ($n_f = \text{const}$). For such problem, the boundary

condition in the spherical coordinates ρ, θ, ϕ reads

$$\varphi(\rho = R_m) = 0. \quad (29)$$

The approximate solution for the potential is given by formula (26). In order to satisfy (29), we need to consider “images” that are reflections of dipoles in the metal. This is the most complicated part in a problems of this kind. In our case, the simplest way to consider the images is, apparently, the following. A real uniformly polarized ball can be represented as two balls with radius R shifted relative each other by the size r_0 of the dipole. The first ball is uniformly charged negatively (so that its total charge is $Q_- = N_f q_0 = Q_0$, where N_f is the total number of initial dipoles in helium). Let its center have the coordinates $x = y = 0$, $z = -r_0/2$. The second ball is charged positively and has the total charge $Q_+ = -Q_-$. The coordinates of its center are $x = y = 0$, $z = r_0/2$. The center of the segment joining both balls is the coordinate origin: $x = y = z = 0$. We consider that these two balls are placed in a metallic sphere so that the $(-)$ and $(+)$ balls touch the internal surface of the sphere. In this case, $R_m = R + r_0/2$. The sphere has contact with the balls at two opposite points, and the remaining points of the sphere are separated from two balls by a thin layer (with a thickness of $\lesssim r_0/2$) of vacuum. The potential created by a uniformly charged dielectric ball with radius R at a point located at the distance R_0 from the ball center is

$$\begin{aligned} \varphi(\mathbf{R}_0) &= \int_V \frac{g dx dy dz}{\varepsilon \sqrt{(x - X_0)^2 + (y - Y_0)^2 + (z - Z_0)^2}} = \\ &= \int_0^R r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \frac{g}{\varepsilon \sqrt{r^2 + R_0^2 - 2rR_0 \cos \theta}} = \frac{2\pi g}{\varepsilon R_0} \int_0^R r dr (r + R_0 - |r - R_0|). \end{aligned} \quad (30)$$

This formula yields

$$\varphi(\mathbf{R}_0) = \begin{cases} \frac{2\pi g}{\varepsilon} \left(R^2 - \frac{R_0^2}{3} \right) & R_0 \leq R, \\ \frac{Q}{\varepsilon R_0} & R_0 \geq R. \end{cases} \quad (31)$$

Here, $g = Q/V = 3Q/(4\pi R^3)$ is the charge density, ε is the dielectric permittivity of the ball. Solution (31) is well known [31]. In this section, we will describe the field created by the initial dipoles immersed in a dielectric. The dielectric weakens the field by ε times. Therefore, the values of charges will be always divided by ε .

All points at a distance of $r \leq R_m - r_0$ from the coordinate origin belong to both balls: $(-)$ -ball and $(+)$ -ball. Let us consider this domain. According to (31), these two balls create at the point \mathbf{r} the potential

$$\varphi(\mathbf{r}) = \frac{2\pi g_-}{\varepsilon} \left(R^2 - \frac{r_-^2}{3} \right) + \frac{2\pi g_+}{\varepsilon} \left(R^2 - \frac{r_+^2}{3} \right). \quad (32)$$

Here, $g_- = Q_0/V$, $g_+ = -g_-$, $V = (4\pi/3)R^3$, and r_- , r_+ are the distances from the point of observation \mathbf{r} to the centers of the $(-)$ and $(+)$ balls, respectively:

$$r_-^2 = r^2 + (r_0/2)^2 - rr_0 \cos(\pi - \theta), \quad (33)$$

$$r_+^2 = r^2 + (r_0/2)^2 - rr_0 \cos \theta. \quad (34)$$

In this case, the vector \mathbf{r} is directed from the coordinate origin to the point of observation, and $\theta = (\widehat{\mathbf{i}_z, \mathbf{r}}) = (\widehat{\mathbf{r}_0, \mathbf{r}})$. Formulae (32)–(34) yield the exact solution

$$\varphi(\mathbf{r}) = -\frac{Q_0 r r_0 \cos \theta}{\varepsilon R^3} = -\frac{Q_0 \mathbf{r} \mathbf{r}_0}{\varepsilon R^3} = \frac{4\pi \mathbf{P}_0 \mathbf{r}}{3\varepsilon}, \quad (35)$$

which is also well known [31]. It is essential that relation (35) follows directly from formula (26). It is easy to see, making use of the relation

$$\int_V d\mathbf{r}_1 \frac{\mathbf{P}_0 \cdot (\mathbf{r} - \mathbf{r}_1)}{\varepsilon |\mathbf{r} - \mathbf{r}_1|^3} = -\frac{\mathbf{P}_0}{\varepsilon} \frac{\partial}{\partial \mathbf{r}} \int_V \frac{d\mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|}.$$

Relation (35) implies that, in the domain $r \leq R_m - r_0$, the electric field strength is constant:

$$\mathbf{E}(\mathbf{r}) = -\nabla_{\mathbf{r}} \varphi(\mathbf{r}) = \frac{Q_0 \mathbf{r}_0}{\varepsilon R^3} = -\frac{4\pi \mathbf{P}_0}{3\varepsilon}. \quad (36)$$

According to (31), the $(-)$ and $(+)$ balls located inside the metallic sphere create on the internal surface of the sphere the potential

$$\varphi(\mathbf{r}) = \frac{Q_-}{\varepsilon r_-} + \frac{Q_+}{\varepsilon r_+}. \quad (37)$$

It coincides with the potential, which is obtained, if the $(-)$ -ball and the $(+)$ -ball are replaced by the point charges Q_- and Q_+ located at the centers of the $(-)$ and $(+)$ balls, respectively. In this case, the boundary condition (29) is easily satisfied: potential (37) can be exactly compensated on the whole surface of the cavity, if we introduce two additional charges that are the images of the point charges Q_- and Q_+ .

Let the point charge Q_- be placed inside a conducting sphere with radius R_m at a distance of $r_0/2$ from its center. It is known (see [24], Chapt. I) that the image of such charge is located at a distance of $l_- = 2R_m^2/r_0$ from the sphere center, and the charge of this image is $q_- = -Q_- 2R_m/r_0$. Moreover, the sphere center, charge, and image are located on the same line, and the charge is placed between the sphere center and the image. The potential created by the charge Q_- and its image is equal to zero on the whole surface of the sphere, which can be directly verified.

The $(-)$ and $(+)$ balls induce polarization charges on the internal surface of the metallic sphere. The field created by these charges inside the sphere coincides with the field of images. With regard for this, the total potential at a point \mathbf{r} inside the sphere is the sum of potentials created at this point by the $(-)$ -ball, $(+)$ -ball, and images of the point charges Q_- and Q_+ . The solution for $r \leq R_m - r_0$ is as follows:

$$\varphi(\mathbf{r}) = -\frac{Q_0 r r_0 \cos \theta}{\varepsilon R^3} + \frac{q_-}{\varepsilon r_{q_-}} + \frac{q_+}{\varepsilon r_{q_+}} = -\frac{Q_0 r r_0 \cos \theta}{\varepsilon R^3} - \frac{2R_m Q_0}{\varepsilon r_0 r_{q_-}} + \frac{2R_m Q_0}{\varepsilon r_0 r_{q_+}}. \quad (38)$$

Here, r_{q_-} and r_{q_+} are the distances from the point of observation \mathbf{r} to the images of the charges Q_- and Q_+ , respectively:

$$r_{q_-}^2 = r^2 + (l_-)^2 - 2rl_- \cos(\pi - \theta), \quad (39)$$

$$r_{q_+}^2 = r^2 + (l_+)^2 - 2rl_+ \cos \theta, \quad (40)$$

where $l_- = 2R_m^2/r_0 = L$ and $l_+ = 2R_m^2/r_0 = L$ are the distances from the image of the charge Q_- and the image of the charge Q_+ to the center of the spherical cavity. Since the charges in (37) are decreased by ε times, the charges of the images in (38) are also decreased by ε times.

For $r_0/R \ll 1$ and $r \leq R_m$, we have $r/L \ll 1$. Let us use r/L as a small parameter. Then relations (39) and (40) yield

$$\frac{1}{r_{q_-}} = \frac{1}{L} \left[1 - \frac{r \cos \theta}{L} + \frac{r^2}{L^2} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) + \frac{r^3}{L^3} \left(\frac{3}{2} \cos \theta - \frac{5}{2} \cos^3 \theta \right) + O\left(\frac{r^4}{L^4}\right) \right], \quad (41)$$

$$\frac{1}{r_{q+}} = \frac{1}{L} \left[1 + \frac{r \cos \theta}{L} + \frac{r^2}{L^2} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) + \frac{r^3}{L^3} \left(\frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta \right) + O\left(\frac{r^4}{L^4}\right) \right]. \quad (42)$$

Substituting expansions (41), (42) in (38) and taking the relation $L = 2R_m^2/r_0$ into account, we get

$$\varphi(\mathbf{r}) = \frac{Q_0 r r_0 \cos \theta}{\varepsilon} \left(\frac{1}{R_m^3} - \frac{1}{R^3} \right) + \frac{Q_0 r^3 r_0^3}{8 R_m^7 \varepsilon} (5 \cos^3 \theta - 3 \cos \theta) + O\left(\frac{r^5}{L^5}\right). \quad (43)$$

In view of the formulae $r r_0 \cos \theta = \mathbf{r} \mathbf{r}_0$, $R_m = R + r_0/2$ and the smallness of r_0/R , relation (43) yields finally:

$$\varphi(\mathbf{r}) = -\frac{Q_0 \mathbf{r} \mathbf{r}_0}{\varepsilon R^3} \frac{3r_0}{2R} + O\left(\frac{r_0^3}{R^3}\right), \quad (44)$$

$$\mathbf{E}(\mathbf{r}) = -\nabla_{\mathbf{r}} \varphi(\mathbf{r}) \approx \frac{Q_0 \mathbf{r}_0}{\varepsilon R^3} \frac{3r_0}{2R}. \quad (45)$$

Two last formulae imply that the spherical conductor decreases potential (35) and the field strength (36) approximately by R/r_0 times. The known mechanisms of polarization of He II give the value of r_0 comparable with (or much less of) the interatomic distance. Taking the realistic values $R \sim 1$ cm and $r_0 \sim 10$ Å, we get $R/r_0 \sim 10^7$. In other words, the *images almost completely suppress the electric field \mathbf{E} inside a spontaneously polarized dielectric ball.*

We now determine the field inside a spherical conductor in a thin layer of width r_0 near the sphere surface. This layer is divided into two regions: the region lying outside the $(-)$ -ball and inside the $(+)$ -ball (or vice versa) and the region lying outside the $(-)$ and $(+)$ balls.

For the region outside the $(-)$ -ball and inside the $(+)$ -ball, the above formulae give

$$\begin{aligned} \varphi(\mathbf{r}) &= \frac{2\pi g_-}{\varepsilon} \left(R^2 - \frac{r_-^2}{3} \right) + \frac{Q_+}{\varepsilon r_+} + \frac{q_-}{\varepsilon r_{q-}} + \frac{q_+}{\varepsilon r_{q+}} \approx \frac{3Q_0}{2\varepsilon R} + \frac{Q_0 \mathbf{r} \mathbf{r}_0}{2\varepsilon R^3} \left(1 - \frac{3r_0}{R} \right) - \\ &- \frac{Q_0 \mathbf{r} \mathbf{r}_0}{2\varepsilon r^3} - \frac{Q_0(4r^2 + r_0^2)}{8\varepsilon R^3} - \frac{Q_0}{\varepsilon r} - \frac{Q_0 r_0^2(3 \cos^2 \theta - 1)}{8\varepsilon r^3} + O\left(\frac{r_0^3}{R^3}\right), \end{aligned} \quad (46)$$

$$\mathbf{E}(\mathbf{r}) \approx -\frac{3Q_0(\mathbf{r} \mathbf{r}_0) \mathbf{i}_{\mathbf{r}}}{2\varepsilon r^4} + \frac{Q_0 \mathbf{i}_{\mathbf{r}}(r^3 - R^3)}{\varepsilon R^3 r^2}, \quad \mathbf{i}_{\mathbf{r}} = \frac{\mathbf{r}}{r}. \quad (47)$$

Since $r \approx R$ for this region, it is seen that the strength \mathbf{E} is comparable by magnitude with strength (36) of the problem without a resonator.

For the region between the dielectric and the conductor (outside the $(-)$ and $(+)$ balls), we have

$$\varphi(\mathbf{r}) = \frac{Q_-}{\varepsilon r_-} + \frac{Q_+}{\varepsilon r_+} + \frac{q_-}{\varepsilon r_{q-}} + \frac{q_+}{\varepsilon r_{q+}} \approx \frac{Q_0 \mathbf{r} \mathbf{r}_0}{\varepsilon R_m^3} - \frac{Q_0 \mathbf{r} \mathbf{r}_0}{\varepsilon r^3}, \quad (48)$$

$$\mathbf{E}(\mathbf{r}) \approx -\frac{Q_0 \mathbf{r}_0}{\varepsilon R_m^3} + \frac{Q_0 \mathbf{r}_0}{\varepsilon r^3} - \frac{3Q_0(\mathbf{r} \mathbf{r}_0) \mathbf{i}_{\mathbf{r}}}{\varepsilon r^4}. \quad (49)$$

Here, the field strength is also comparable with (36). In this case, strength (47) is directed radially, (49) has the radial and z components, and strength (36) is directed along the z -axis.

In the region $r \leq R_m$, the potential is continuous, whereas the field strength undergoes a jump on the surface of the dielectric and the internal surface of the conductor. It is seen from (48) that the boundary condition (29) is satisfied.

Thus, the electric field is strong (comparable with the field in the absence of a conductor) only in the narrow space of thickness $\leq r_0$ near the internal surface of the conductor. In the remaining volume inside the conductor (i.e., in almost whole volume of the dielectric), the field \mathbf{E} is almost completely suppressed. The effect is related to the presence of the spherical conductor around the dielectric and, therefore, can be called the “spherical blackout”. This effect has a simple explanation. Without a conductor, the field \mathbf{E} inside the dielectric is uniform (see (36)). The field created by the conductor in the region $r \leq R_m$ coincides with the field of images. But the images are remote from the dielectric by the distance $L = 2R_m^2/r_0$, which is much larger than the size $2R$ of the dielectric. Therefore, the field created by the images inside the dielectric is almost uniform, but is directed against the intrinsic field (36) of the dielectric and compensates it. If the compensation would be absent, then the condition $\varphi = 0$ would not be satisfied: the condition $\varphi = \text{const}$ requires that the tangential component (\mathbf{E}_t) of the field \mathbf{E} on the internal surface of the conductor be zero. Since the field \mathbf{E} is uniform inside the dielectric, the condition $\mathbf{E}_t = 0$ on the surface requires that $\mathbf{E} \approx 0$ in the whole volume of the dielectric. This is a purely electrostatic effect related to the geometry of the system, but not to low temperatures.

If we surround a system of charges by a conductor, it causes a redistribution of the equipotential surfaces so that these surfaces near the conductor be parallel to its surface. But if the conductor is grounded, then $\varphi = 0$ in the conductor and the potential is suppressed near the conductor. In the above-considered problem, several factors act simultaneously, and, therefore, the conductor suppresses the potential in almost whole internal volume. If our sphere is not grounded, then $\varphi = \varphi_0 = \text{const}$ on the boundary. In this case, the term φ_0 should be added to solutions (44), (46), and (48) for the potential, but \mathbf{E} is not changed.

In the calculation of the effect, we replaced the exact formulae (19) by the approximate one (26). But since the effect has a symmetry nature, we expect that this effect will hold in the calculation with exact Eqs. (19) as well.

Since He II is in the gravity field, helium is always spontaneously polarized. If the walls of a vessel are vertical, and the bottom is horizontal, the polarization is uniform. In this case, the electric field is weak, but is measurable [10]. However, if the shape of a vessel is a sphere, then the polarization should have the z - and \mathbf{r} -components and, therefore, should be nonuniform. In other words, it is apparently impossible to get the uniform spontaneous polarization in a spherical vessel. May be, the black-out effect can be realized in experiments with some different configuration of the field.

We note that a decrease in the electric energy of the dielectric because of the location of the dielectric in a spherical conductor is equal to the work (with opposite sign) that must be executed in order to transport two metallic hemispheres from infinity and to enclose a polarized dielectric by these hemispheres.

According to the above-presented solutions, the boundaries affect strongly such bulk property of a macroscopic system as the electric field. This is not typical of great systems and is related, in our case, to the long-range character of the Coulomb interaction.

4 Analysis of the experiments by Rybalko and Chagovets

T. S. Chagovets [3] has repeated the Rybalko’s experiment [1] and has obtained the value of $\Delta U/\Delta T$, which is larger by two orders of magnitude. It is necessary to understand this large

difference. The reason for such difference can be instrumental or physical. It was assumed [3] that the difference is due to the input capacity C_{in} , because $\Delta U = \Delta Q/C_{in}$. In fact, this means that the difference of the results of works [1] and [3] arose due to the different calibration of voltmeters. We do not know all details of measurements, but it is clear that the quantities ΔU and ΔT should be measured on the common base. The construction of a detailed theory makes no sense, if the experimental error is so large. It is quite obvious qualitatively that He II can be polarized by several mechanisms.

Let us discuss possible physical reasons. As is known, both surface and bulk electric fields contribute to the signal ΔU . Let the bulk signal be dominant. In the experiment [1], the resonator was grounded and covered inside with a metal. Under the complete grounding, the boundary condition is as follows: $\varphi = 0$ on the internal surface of the resonator. Near the resonator wall, the equipotential surfaces are parallel to this wall. In the experiments [1], the electrode was placed in parallel with the resonator end at the distance $\delta = 0.05\text{mm}$ from the latter [32]. Between the electrode and the resonator, the dielectric with $\varepsilon = \varepsilon_2 \approx 9.3$ was placed [32]. We note that the electrode moves the equipotential surfaces away from one another. Therefore, the measured quantity ΔU is the difference between the potential in helium (without electrode) at the point, where the electrode is placed, and the potential ($\varphi = 0$) at the nearest point of the resonator. The quantity δ should be additionally divided by $\varepsilon_2/\varepsilon_{He}$, since the distance between equipotential surfaces in the support-dielectric is larger by $\varepsilon_2/\varepsilon_{He}$ times. Therefore, $\Delta U \sim \varphi_0[(\delta \cdot \varepsilon_{He}/\varepsilon_2)/0.5L]^k$, where $\varphi_0 \sim L\Delta T \cdot \text{const}$ is the amplitude of oscillations of the potential at the resonator center, and $k \simeq 1-2$. Such estimate follows from that the function $\varphi(\rho, z)$ should have the shape of a dome with the “tip” at the point $\rho = 0, z = 0$ (resonator center), since such shape is inherent in a second-sound half-wave, and since $\varphi = 0$ on the boundary. In the experiment by Chagovets [3], the resonator is a dielectric with $\varepsilon = \varepsilon_3$. Therefore, it will decrease φ on the boundary not down to zero, but by $\sim \varepsilon_3$ times. In this case, $\Delta U \sim \varphi_0 \varepsilon_{He}/\varepsilon_3$. The resonator length L in [3] is close to the length L of a long resonator in [1]. Therefore, the values of φ_0 should also be close. We get that ΔU from [3] should be larger by $\zeta \sim [0.5L\varepsilon_2/\delta]^k/\varepsilon_3$ times than ΔU in the experiment by Rybalko. Since $\varepsilon_2 \sim \varepsilon_3$, we have $\zeta \gtrsim L/\delta \gtrsim 10^2$. In other words, the bulk signal in the experiment [3] should be higher by several orders of magnitude than in the experiment by Rybalko. The reason consists in that a dielectric suppresses the potential near the walls less, than the conductor.

However, the values of $\Delta U/\Delta T$ in [1] were the same for two resonators with different L and identical δ . It is clear from the above consideration that the values of $\Delta U/\Delta T$ for the bulk signal should be different in this case. Hence, the signal was a surface one. This is also supported by the fact that $\Delta U(z) = \text{const} \cdot \Delta T(z)$ [2]: for such proportionality, $\Delta U(z)$ is defined by local properties of the medium, but the bulk signal is formed by the remote points as well, according to (26). The experimental dependence $\Delta U(z)$ [2] is in good agreement with the theoretical result for a surface signal [15]. And the principal point is the following: in the experiments [2, 4], the dielectric resonator was used. If the signal would be bulk one, then it should be significantly larger in [2, 4] than in [1]. But the signals were close. Therefore, we are sure that the surface contribution to the signal was dominant in [1, 2, 4].

Consider the experiment [3], in which ΔU between the metallic end and the electrode glued to it was measured. Glue is a dielectric. The temperature expansions-compressions of the electrode in a second-sound wave cause the periodic compression of glue. For this reason, the polarization should arise in glue periodically. Therefore, it is possible that the signal [3] is the signal from glue. We propose to measure $\Delta U/\Delta T$ with the same resonator, but with a different glue. One can also to insulate the electrode from the end by a thick

dielectric (like [4]). Note that ΔU in the experiments [1, 2, 4] was measured between the electrode and the ground. The conditions in [1, 2, 4] were different, but the signals $\Delta U/\Delta T$ were close. Therefore, it should be the signal from helium.

Moreover, the distinction of $\Delta U/\Delta T$ in [3] from the result in [1, 2, 4] can be caused by admixtures. As ^4He is fluidized, the admixtures settle to walls. But a small fraction can remain, possibly, in the bulk and can settle to the electrode. A small amount of extrinsic polar molecules can suppress the bulk signal and can change strongly the surface one [15]. The admixture of ^3He can increase the signal by 1-2 orders of magnitude [15]. Therefore, we propose to carry out the experiment with ^4He purified from admixtures. Molecules and ions of air can also settle to the electrode (before its immersion in He II). But since the temperature of air is high, the number of such particles should be very small. Otherwise, the signal ΔU would be different at different laboratories.

In our opinion, it is of interest to carry out the experiments like those in [1, 2, 3, 4], by adding the admixture of ^3He in ^4He [15]. Metals can (1) be nonwetable by ^4He , (2) be wettable, and (3) strongly attract ^4He (in this case, the first layer of ^4He near a metal is solidified) [33, 34, 35, 36]. In the experiments [1, 2, 3, 4], the electrodes made of metals only of group 3 were used. It would be worth to apply the electrodes made of metals of the first or second group, then the surface signal should be strongly changed [15]. We believe that the purposeful variation of the experimental conditions will allow one to clarify the role of various factors.

5 Conclusion

We have constructed a method of description of the spontaneous polarization of a dielectric. It is based on the Maxwell equations for a medium and the statistical averaging for an ensemble of dipoles. We have also calculated the electric field in a spontaneously polarized dielectric surrounded by a spherical conductor with the exact account for the boundary condition $\varphi = 0$ on the surface of the conductor. It turns out that, under the uniform initial polarization of the dielectric, the conductor suppresses the electric field \mathbf{E} inside the dielectric.

In addition, we have discussed the possible reasons for the distinction of the results in the experiments by Rybalko [1] and by Chagovets [3]. We note that a second-sound wave is accompanied by many weak electric processes (including those in the resonator frame), which are synchronous with the second sound. Therefore, the clarification of the nature of the effect will require a series of experiments and more exact calculations.

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